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From N=2 Strings to F & M Theory [1], [2]

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Abstract

Taking the N=2 strings as the starting point, we discuss the equivalent self-dual field theories and analyse their symmetry structure in $2 + 2$ dimensions. Restoring the full 'Lorentz' invariance in the target space necessarily leads to an extension of the N=2 string theory to a theory of $2 + 2$ dimensional *supermembranes* propagating in $2 + 10$ dimensional target space. The supermembrane requires *maximal* conformal supersymmetry in $2 + 2$ dimensions, in the way advocated by Siegel. The corresponding self-dual N=4 Yang-Mills theory and the self-dual N=8 (gauged) supergravity in 2+2 dimensions thus appear to be naturally associated to the membrane theory, not a string. Since the same theory of membranes seems to represent the M-theory which is apparently underlying the all known N=1 string theories, the N=2 strings now appear on equal footing with the other string models as particular limits of the unique fundamental theory. Unlike the standard 10-dimensional superstrings, the N=2 strings seem to be much closer to a membrane description of the F & M theory.

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1 Introduction

Since the discovery of string dualities, much evidence was collected in favor that various 'different' string theories can be understood as particular limits of a unique underlying theory whose basic formulation is yet to be found. It seems also that the fundamental theory is not just a theory of strings but it describes fields, strings and membranes in a unified way. There is a candidate for such unified theory – the so-called *M-theory* [1, 2] [4] or its refined *F-theory* formulation [4]– which can be reduced to all known 10-dimensional supertrings and 11-dimensional supergravity as well. Accordingly, there should be a similar way to understand strings with the *extended* world-sheet supersymmetry – the so-called N=2 strings – in terms of the M-theory.

As was noticed recently [5], the $N = (2,1)$ heterotic string is not only connected to the M-theory in particular backgrounds, but it also suggests the M-theory definition as a theory of $2 + 2$ dimensional membranes (sometimes called *M-branes* [6]) embedded in $2 + 10$ dimensions with a null reduction. If so, the origin of M-branes should be understood from the basic properties of N=2 strings. It is the purpose of this paper to argue that the hidden world-volume and the membrane target space dimensions are in fact *required* by natural symmetries which are broken in the known N=2 string formulations. By the natural symmetries I mean the 'Lorentz' invariance and supersymmetry in $2 + 2$ dimensions. These symmetries also uniquely determine the dymanics of M-branes which is given by the self-dual gauged supergravity with the maximally extended N=8 supersymmetry. The relevant maximally extended self-dual field theories were constructed some time ago by Siegel [7] in the light-cone gauge (see also ref. [8]) but, unlike the earlier expectations, they appear not to be related to the N=2 strings, but to the M-branes. The suggestion that the M-branes should be described by a kind of self-dual gravity coupled to a self-dual matter also appeared in ref. [5]. It is the goal of this paper to specify the symmetries of this self-dual field theory. Unlike the analysis of the $1+1$ and $1+2$ dimensional target space versions of the $N = (2,1)$ strings in ref. [5], we impose the $2+2$ dimensional 'Lorentz' symmetry as the crucial symmetry of M-branes.

2 N=2 string symmetries

The N=2 strings are strings with two world-sheet (local) supersymmetries. There exist $N = (2,2)$ open and closed strings, and $N = (2,1)$ and $N = (2,0)$ heterotic

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[4]See ref. [3] for a review.

strings. The N=2 strings have twenty years-long history. [5] The gauge-invariant $N = 2$ string world-sheet actions in the NSR-type formulation are given by couplings of a two-dimensional N=2 supergravity to a complex N=2 scalar matter [13]. Gauge-fixing produces conformal ghosts, complex superconformal ghosts and real abelian ghosts, as usual. The corresponding (chiral) world-sheet current algebras include a stress-tensor, two supercurrents and an abelian current; taken together, they constitute an N=2 superconformal algebra. As a result, the critical closed and open N=2 strings live in four dimensions with the signature $2 + 2$. [6] The current algebras of the N=2 heterotic strings include an additional abelian null current needed for the nilpotency of the BRST charge, and it effectively reduces the target spacetime dynamics down to $1+2$ or $1+1$ dimensions [14]. The full target space dimension (with the heterotic modes) is $2 + 26$ for the left-moving modes of the $N = (2, 0)$ heterotic string and $2 + 10$ for the $N = (2, 1)$ ones, respectively.

The BRST cohomology and on-shell amplitudes of N=2 strings were investigated in detail by several groups [14, 15, 16, 17]. The results of that investigations can be summarized as follows. There exist only one massless particle in the open or closed N=2 string spectrum. This particle can be identified with the Yang scalar of self-dual Yang-Mills theory for open strings, or the Kähler scalar of self-dual supergravity for closed strings, while infinitely many massive string modes are all unphysical. The NS- and R-type states are connected by the spectral flow, which is also a symmetry of correlation functions. Accordingly, the N=2 strings lack 'space-time' supersymmetry. Twisting the N=2 superconformal algebra yields some additional twisted physical states which would-be the target space 'fermions', but they actually decouple. The corresponding 'space-time fermionic' vertex operators anticommute modulo picture-changing, instead of producing 'space-time' translations, as required by 'space-time' supersymmetry [16]. The only non-vanishing scattering amplitudes appear to be 3-point trees (and, maybe, 3-point loops as well), while all the other tree and loop N=2 string amplitudes apparently vanish due to kinematical reasons. As a result, an N=2 string theory appears to be equivalent to a self-dual field theory. In particular, the N=2 open string amplitudes are reproduced by either the Yang non-linear sigma-model action [18] or the Leznov-Parkes cubic action [19], following from a field integration of the self-dual Yang-Mills equations in particular Lorentz non-covariant gauges, and related to each other by a duality transformation. As far as the closed N=2 strings are concerned, the equivalent non-covariant field theory action is known as the Plebanski action [20] for the self-dual gravity.

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[5]See ref. [9] for the first references on the subject, and refs. [10, 11, 12] for a review.

[6] The signature is dictated by the existence of a complex structure and non-trivial kinematics.

One generically finds more massless physical states in the heterotic N=2 string spectra. In particular, the $(2,1)$ heterotic string has 8 bosonic and 8 fermionic massless particles in $1+2$ dimensions. The equivalent field theory is given by a three-dimensional coupling of self-dual Yang-Mills and self-dual gravity [14]. Unlike the N=0 and N=1 strings, the N=2 string world-sheet symmetries do not allow massive string excitations to be physical.

The natural global continuous ('Lorentz') symmetry of a flat $2+2$ dimensional target space ('space-time') is $SO(2,2) \cong SU(1,1) \otimes SU(1,1) \cong SL(2,\mathbf{R}) \otimes SL(2,\mathbf{R})$. The NSR-type N=2 string actions used to calculate the N=2 string amplitudes have only a part of it, namely, $U(1,1) \cong SU(1,1) \otimes U(1)$ or $GL(2,\mathbf{R})$, so is the symmetry of amplitudes in the absence of world-sheet abelian instantons. [7]

Adding to the N=2 string generators of the N=2 superconformal algebra the spectral flow operator and its inverse provides the raising and lowering operators of $SU(1,1)$. Closing the algebra, one extends the initial N=2 superconformal algebra to the 'small' twisted N=4 superconformal algebra. This remarkable property allows one to treat the N=2 string theory as an N=4 topological field theory [15]. [8] It is even more important that this reformulation brings the additional internal symmetry, $SU(1,1)$, which is just needed to restore the broken 'Lorentz' symmetry $U(1,1)$ to the $SO(2,2)$. The embeddings of the N=2 algebra into the N=4 one are parameterized by vielbeins (twistors) belonging to the harmonic space $SU(1,1)/U(1)$, which is the space of all complex structures in $2+2$ 'space-time'. The harmonic space technically adds two additional world-sheet dimensions to an N=2 string. It now becomes obvious that, in order to get back the 'Lorentz' symmetry in the N=2 string theory target space, one has to take the two additional harmonic (one time-like and another space-like) dimensions for real, by *extending* the N=2 string world-sheet to the $2+2$ dimensional world-volume (M-brane), i.e. to complexify the N=2 string world-sheet coordinates τ and σ. [9]

The target space for M-branes, where they are supposed to propagate, can also be fixed by merely restoring the 'space-time Lorentz' symmetry of the N=2 strings, as we are now going to argue.

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[7] Taking into account N=2 string amplitudes with a non-trivial Chern class (or $U(1)$ instanton number) makes the symmetry even lower [16].

[8] It does not, however, mean that the critical N=4 strings are 'the same' as the critical N=2 strings.

[9] The idea of the string world-sheet complexification also appeared in the stidies of high-energy behavior of string theories [21].

3 $2+10$ **out of** $2+2$

It was first noticed by Siegel [7] that the self-duality and 'Lorentz' invariance in $2+2$ dimensions imply the *maximal* supersymmetry. It becomes transparent in the light-cone gauge for self-dual field theories, where only physical degrees of freedom are kept.

The irreducible massless representations of $SO(2,2) \cong SL(2,\mathbf{R})' \otimes SL(2,\mathbf{R})$ are either self-dual or chiral, and they are all real and one-dimensional. It is therefore convenient to introduce the basis, in which a $2+2$ dimensional vector has components $x^{\alpha',\alpha}$, so that the first helicity index $\alpha' = (+', -')$ refers to the first $SL(2,\mathbf{R})'$ component while $\alpha = (+,-)$ to the second one. In this basis, $x^{\alpha',-}$ are treated as 'time' coordinates, whereas $x^{\alpha',+}$ as 'space' coordinates. In the light-cone gauge, a self-dual gauge theory is described in terms of a single field – the so-called *prepotential* $V(x)$ – whose helicity is $+1$ for the self-dual Yang-Mills theory ($V_=$), and is $+2$ for the self-dual gravity ($V_{==}$). Note that going to the light-cone gauge already breaks the 'Lorentz' invariance down to $SL(2,\mathbf{R})' \otimes GL(1)$. The N-supersymmetrization of the light-cone gauge description of a self-dual gauge theory is straightforward: one should simply extend the prepotential to an N-extended real chiral superfield $V(x, \theta^+)$ to be also dependent on N anticommuting (Grassmannian) self-dual superspace coordinates $\theta^{A,+}$, $A = 1, 2, \ldots, N$, which are Majorana-Weyl spinors in $2+2$ dimensions.

A free field theory action in the light-cone gauge takes the universal form,

$$I_{\text{free}} = \tfrac{1}{2} \int d^{2+2}x \, d^N \theta^+ \, V_{-,\ldots} \Box V_{-,\ldots} \, , \tag{1}$$

and it simultaneously determines the field content of the theory. For generic N, the I_{free} is not 'Lorentz'-invariant, but it becomes invariant when the light-cone superfield V is self-dual or, equivalently, when the $GL(1)$ charge of the superspace measure cancels that of the integrand in eq. (1). Both requirements obviously imply $N = N_{\text{max}}$. For example, the first component of the maximally extended N=4 supersymmetric self-dual Yang-Mills prepotential $V_=$ has the helicity $+1$, whereas its last component has helicity -1, which is just needed for a 'Lorentz'-invariant action. Similarly, one finds that the free action (1) in terms of the N-extended self-dual supergravity prepotential $V_{==}$ requires the $N = 8$ supersymmetry in order to be 'Lorentz'-invariant. Siegel gave the light-cone formulations for the maximally supersymmetric self-dual gauge and gravity field theories, both in components and in self-dual superspace [7]. The full (interacting) theories he constructed are actually quite similar to the non-self-dual supersymmetric gauge theories to be formulated in the light-cone gauge [22].

As far as the heterotic N=2 'strings' are concerned, in the $N = (2, 0)$ case one gets the N=4 supersymmetric coupling of self-dual super-Yang-Mills to self-dual supergravity. In the $N = (2, 1)$ case, [10] the 'Lorentz' invariance still requires N=8, while the gauge group for the heterotic vector bosons is obviously restricted to $SO(8)$ or its non-compact version.

The appearance of the $SO(8)$ internal symmetry in the maximally supersymmetric heterotic case is quite remarkable. Having substituted N=2 strings by M-branes, we thus restored not only the 'Lorentz' $SO(2, 2)$ symmetry but the $N = 8$ supersymmetry and the $SO(8)$ internal symmetry too. We are now able to proceed in the usual way known in supergravity, and 'explain' the maximally extended supersymmetry as a simple supersymmetry in twelve dimensions,

$$SO(2, 2) \otimes SO(8) \subset SO(2, 10) .$$

Note that the $2 + 10$ dimensions are the nearest ones in which the Majorana-Weyl spinors and self-dual tensors also appear, like in $2 + 2$ dimensions. It should be noticed that twelve dimensions for string theory were originally motivated in a very different way, namely, by a desire to explain the S-duality of type IIB string in ten dimensions as the T-duality of the 12-dimensional string dimensionally reduced on a two-torus [3]. The type IIB string is then supposed to arise upon double dimensional reduction from the hypothetical $2 + 10$ dimensional F-theory.

4 M-branes and their symmetries

Since the full covariant $2+2$ dimensional action describing M-branes is still unknown, the first step towards its construction is to determine the world-volume and target-space symmetries it should possess. Since in the light-cone gauge it is supposed to describe the self-dual gauged $N = 8$ supergravity, [11] and the latter actually possess the larger *superconformal* symmetry $SL(4|8)$ [7], it should also be the fundamental world-volume symmetry for the M-branes. It simply follows from the facts that the conformal extension of the 'Lorentz' group $SO(2, 2)$ is given by the group $SL(4) \cong SO(3, 3)$, while its N-supersymmetric extension is the superconformal group $SL(4|N)$. Accordingly, one should use the symmetry $SSL(4|4)$ for the 'open' M-branes. The

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[10]It was suggested [5] that the $N = (2, 1)$ heterotic strings describe the strong coupling dynamics of the ten-dimensional superstrings compactified down to two dimensions.

[11]One may distinguish between the 'closed' and 'open' M-branes corresponding to the maximal $N = 8$ and $N = 4$ world-volume supersymmetry, respectively.

six dimensions which are known to be distinguished by the string-string duality, are also distinguished for describing M-branes since the $2+2$ dimensional superconformal group has a unique *linear* representation only in six dimensions. Since the internal symmetry of the $N = 4$ superconformal group is $SL(4) \cong SO(3,3)$, combining it with the $2+2$ dimensional conformal group $SO(3,3)$ also implies 'hidden' twelve dimensions in yet another way: $SO(3,3) \otimes SO(3,3) \subset SO(6,6)$. The $6+6$ dimensions is the only alternative to $2+10$ dimensions where Majorana-Weyl spinors also exist.

It should be noticed that we didn't recover the full $SO(2,10)$ symmetry, which is the natural 'Lorentz' symmetry in $2 + 10$ dimensions, but some of its natural decompositions. It may be related to the fact that there is no covariant supergravity theory in $2 + 10$ dimensions. Twelve dimensions may however be useful as a book-keeping device at least. A natural way to deal with the non-covariance problem may be to employ a null reduction [14, 5] which effectively reduces the target space of M-branes down to $1 + 10$ dimensions, thus making a contact to the 11-dimensional supergravity and M-theory. It is also worthy to mention that the group $SO(2,10)$ is the conformal group for $1 + 9$ dimensions.

5   Conclusion

Our arguments support the idea [5] that the unifying framework for describing the M-theory is provided by the $2+2$ dimensional supermembrane theory in $2+10$ dimensions. Self-duality of membranes naturally substitutes and generalizes the conformal symmetry of string world-sheet. Our basic symmetry requirements were merely the linearly realised 'Lorentz' invariance and 'space-time' supersymmetry in $2 + 2$ dimensions. A new feature is the presence of the maximal world-volume conformal supersymmetry. Although this conformal symmetry is non-linearly realized in $2 + 2$ dimensions, there exists its linear realization in six dimensions. The target space dimension (12) is maximal in the sense that it it accommodates all known supergravity theories, as well as the maximal number (8) of Majorana-Weyl spinor supercharges.

The superconformal symmetries of M-branes should be responsible for their full integrability and the absence of UV divergences in $2 + 2$ dimensions. Even though a four-dimensional M-brane action is expected to be non-linear and, hence, non-renormalisable as a quantum theory, it may still be UV finite. For example, the maximally supersymmetric DNS non-linear sigma-model is likely to be UV finite in $2 + 2$ dimensions, as was recently argued in ref. [23]. The M-brane theory should actually have an even larger underlying symmetry given by an *affine* extension of the

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(super)conformal symmetry, which is known to be hidden in the DNS theory [24], and in self-dual field theory equations as well [25].

Our way of reasoning unifies all strings and superstrings with any number of world-sheet supersymmetries towards the M-theory. It seems to be the good alternative to the 'conformal' embeddings proposed earlier [26]. All string theories now arise by combining a compactification of the M-brane with a GSO projection.

Unlike the N=2 strings facing severe infra-red problems in loop calculations [27], there are no such problems for M-branes due to the higher world-volume dimension and supersymmetry. The theory of M-branes should therefore exist as a quantum theory, in which strings would appear as asymptotic states of M-brane.

Acknowledgement

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